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Transforming the Time Scale in Linear Multivariate Growth Curve Models

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Abstract: Latent growth curve models represent repeated measures of outcome variables as functions of consecutive time points and other measures. Already a few authors noticed that the relationship between the initial status and growth rate depends on the time scale involved in the model. Different time scales lead to different estimates of these two growth parameters, as well as their variances and covariances. In this article we consider the multivariate growth curve model, in which the relationship between patterns of change of more than one outcome variable can be modeled. We will show that the dependency also occurs in the multivariate case. Mathematical evidence will be presented in which the relationship will be established of initial status and growth rate with the selected time scale. The nature of the relationship will be illustrated on models with a different time scale for the same empirical data.

Introduction

The multivariate growth curve model is a straightforward extension of the univariate growth curve model, which represents repeated measures of a specific outcome variable as a function of consecutive time points and other measures (Duncan & Duncan, 1995; Willet & Sayer, 1994). Multivariate growth curve models provide additional information on the relationship between patterns of change of different outcome variables. The values of an individual on the outcome variables at a specific time point are modeled as a function of the

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underlying linear growth process. The parameters of such a process (the initial status and the growth rate) vary randomly across individuals, and they are allowed to covary within each process as well as between the processes. Questions about intra- and interindividual variation on multiple outcome variables can be investigated, especially with regard to covarying patterns of change on different variables across individuals (Maccallum, Kim, Malarkey & Kiecolt-Glaser, 1997).

The correlation between the growth parameters, initial status and growth rate, has been a central parameter in the analysis of growth models. By including correlates and predictors of the individual growth parameters it also becomes possible to study systematic differences in the individual growth processes (Willet & Sayer, 1994). Parameters of one growth process can be used in a multivariate growth model as predictors of the growth parameters of another growth process, and vice versa. The extension of univariate latent growth curve methodology to multivariate models provides a commonly accepted framework for the analysis of multivariate longitudinal data. Substantive questions, which can be examined using a multivariate growth curve model, are questions like: Is there evidence for systematic change and individual variability in change in the outcome variables? Are time invariant predictors (e.g. age and gender) related to the initial levels or growth rates? Is the growth rate of one process related the growth rate of another process? Do earlier level of an process predict later growth rates of another process? It is not well documented, however, that the conclusions drawn from growth curve analysis are extremely sensitive to the selected time scale. The scale metric chosen for the time factor has serious consequences for the substantive conclusions based on the model. Within the same set of data, different values of the basis function for the time factor (Meredith & Tisak, 1990, p. 108) lead to different relationships between the initial status and growth rate, as is shown already by several researchers (Garst, 2000; Mehta & West, 2000; Rogosa & Willet, 1985; Rovine & Molenaar, 1998; Rudinger & Rietz, 1998). Stoel & Wittenboer (2000) investigate the relationship for univariate growth curve models with effects of an exogenous predictor on the initial status and growth rate.

In this paper we investigate the relationship in the multivariate case. We will show how the relations between the growth characteristics of different growth processes also depend on the metric of the selected time scale involved. Related ideas can already be found in Rogosa and Willet (1985), and have recently been investigated by Garst (2000) and Stoel and Wittenboer (2000).

The paper is build up as follows. Section 2 starts with a brief introduction into univariate and multivariate latent growth curve models and it illustrates the ideas with a relatively simple example. In Section 3 we formally derive how relations between growth parameters of different growth processes depend on the metric of the time scale used. The fourth section illustrates the ideas with an empirical example. The paper ends up with a discussion in which implications and possible solutions are discussed.

Time and the (Multivariate) Linear Growth Curve Model

The first demonstration that different selections of the time for the initial status lead to different correlations in the same data between initial status and growth rate (ranging from significantly positive to significantly negative) was given by Rogosa and colleagues (Rogosa, Brandt & Zimowski, 1982; Rogosa & Willet, 1985). Their conclusion that, no such thing as the correlation between initial status and change exists, was quite surprising, because determination of a unique correlation seemed to be the goal of much empirical research (Rogosa & Willet, 1985, p. 225).

Although the conclusion of Rogosa and Willet (1985) was obvious and even though it was elaborated further in other areas like regression analysis (Aiken & West, 1991), it took some time before the ideas got ground in latent growth curve methodology. Only recently, renewed interest in time dependency of growth parameters in growth curve modeling has emerged (Garst, 2000; Mehta & West, 2000; Rovine & Molenaar, 1998; Rudinger & Rietz, 1998; Stoel & Wittenboer, 2000).

Point of departure will be a relatively simple multivariate growth curve model with two outcome variables, so that two growth processes are modeled simultaneously. The growth parameters of the First growth process are modeled here as exogenous variables with an effect on the parameters of the second growth process. This model contains most of the difficulties involved in varying the time scale, so parameter changes due to different time scale metrics can be illustrated quite well. Extensions to multivariate models with reciprocal effects between the growth parameters of the two growth processes are straightforward, but will unnecessarily complicate the model. Further details about latent growth curve models can be found in Maccallum et al. (1997), Meredith & Tisak (1990), and Willet & Sayer (1994). If the multivariate growth curve model is specified as a so-called `füll LISREL model', combining the x-side and y-side of the LISREL model, the cross-process effects can be modeled in the Γ matrix of the model. This specification has the advantage that it will greatly simplify the exhibition in this paper because one does not have to compute the matrix $\mathbf{(I\text{-}B)}^{-1}$. On the other hand, specifying the multivariate growth curve model entirely on the y-side allows reciprocal effects between the growth parameters to be estimated. The model can be expressed, in matrix form, as a normal confirmatory factor analysis model:

$$X = \tau_X + \Lambda_X \xi + \delta \tag{1}$$

$$y = \tau_y + \Lambda_y \eta + \varepsilon \qquad (2)$$

where **x** and **y** are the *i x 1* vectors of observed repeated measures, while τ_X and τ_y are the i x 1 vectors of item intercepts. Λ_x, and Λ_y are the i × m matrices of basis function coefficients linking the latent growth parameters to the observed variables, ξ and η are the m × 1 vector of latent random growth parameters, and δ and ε are the i × 1 vectors of random disturbances. As usual, vectors τ_X and τ_y are constrained to zero (see Rovine & Molenaar, 2000; Willet & Sayer, 1994, p. 369) and both Λ_x and Λ_x are (partly) constrained to known time points to incorporate time into the growth model. Alternatively, Equation 1 and 2 can be thought of as single univariate growth models. The equation for the structural part of the model for the latent growth parameters can be expressed as:

$$\eta = \alpha + \Gamma \xi + \zeta \qquad (3)$$

where α is the m × 1 vector of latent means, Γ is the *m* × *m* coefficient matrix for the latent growth parameters, ζ is the m × 1 vector of error in the equations or random disturbances, and E (ξ) = κ.
The covariance matrix of this model (Equations 1 to 3) is:

$$\Sigma = \begin{pmatrix} \Lambda_y (\Gamma\Phi\Gamma' + \Psi)\Lambda'_y + \Theta_\varepsilon & \Lambda_y \Gamma\Phi\Lambda'_x \\ \Lambda_x \Phi\Gamma'\Lambda'_y & \Lambda_x \Phi\Lambda'_x + \Theta_\delta \end{pmatrix} \qquad (4)$$

where Θ_δ and Θ_ε are the covariance matrices of respectively δ and ε; Φ and Ψ are the covariance matrices of respectively ξ and η Standard SEM assumptions are made (see Jöreskog & Sörbom, 1996, p. 2; Bollen, 1989); together with the following additional assumptions. (1) Linear growth for the time interval of interest. (2) `Time-structured' data (Bock, 1979): both the number and the spacing of the assessments must be the same for all subjects (Willet & Sayer, 1994, p. 365). (3) Each individual latent factor score can be expressed as the latent factor mean plus a latent deviation score from the mean (Duncan et al., 1999). (4) The assumption of common causation: the sources of between-occasion variation and individual differences are identical (Mandys, Dolan & Molenaar, 1994).

If the basis function coefficients in Λ_x, and Λ_y are left free to estimate, Equations 1 and 2 represent the more general growth model (Meredith & Tisak, 1990) that also allows for non-linear growth curves. Linear growth, however, requires certain parameters in the basis function to be constrained to specific values. To obtain the linear growth curve model, constraints have to be placed on the basis function coefficients (see Figure 1 and Equations 5 and 6). If the growth rate factors represent a 'unit of change', then each basis function coefficient represents the change that occurs between that occasion and the origin of the process (Rovine & Molenaar, 1998).

The graphic form of a multivariate linear latent growth curve model with 4 consecutive measures of two outcome variables x and y appears in Figure 1. It shows a multivariate growth model in which the parameters of one growth process are used as predictor of the parameters of another growth process. In the example, to be presented in the next paragraph, we will elaborate further on this specific model. Constraining the Basis coefficients to specific values, give η_0 and ξ_0 respectively η_1, and ξ_1 the interpretation of initial status and growth rate. So, η_0 and ξ_0 represent the expected status at the start of each process, and η_1 and ξ_1 designate the average true growth rates. In particular, the basis function coefficients for the initial status factors η_0 and ξ_0 are constrained to 1 and the Basis function coefficients for the growth rate factors η_1 and ξ_1 are constrained to 0, 1, 2 and 3, respectively.

The covariance between initial Status and growth rate has been modeled in the picture, as well as effects between the two linear growth processes.

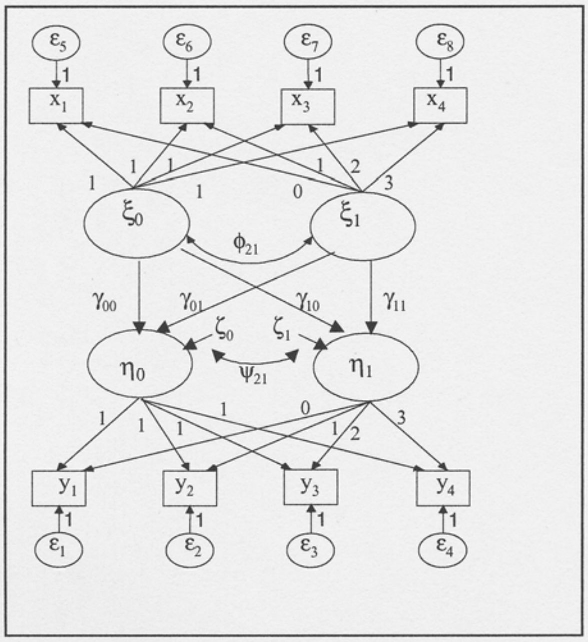

Figure 1. Multivariate linear growth model with constrained basis functions.
Note: Only the covariance structure is shown in this model

The matrix specification corresponding to Equations 1 and 2 is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & t_{1x} \\ 1 & t_{2x} \\ 1 & t_{3x} \\ 1 & t_{4x} \end{bmatrix} \begin{bmatrix} \xi_0 \\ \xi_1 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & t_{1y} \\ 1 & t_{2y} \\ 1 & t_{3y} \\ 1 & t_{4y} \end{bmatrix} \begin{bmatrix} \eta_0 \\ \eta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix} \tag{6}$$

A natural origin of time is a mere coincidence in most social and behavioral research. Therefore, the interpretation of the initial status is established usually at an arbitrary point in time (see also Garst, 2000, p. 61, Willet and Sayer, 1994, p. 367). In Figure 1, the origin of both processes is defined at the first measurement occasion by constraining the first basis function coefficients to zero ($t_{1x} = 0$ and $t_{1y} = 0$). If the basis function coefficients of the first measurement occasion had been constrained to values other than zero, the origin of the processes would have been established at another point in time.

Assume, for instance, that our model describes the development of assertiveness and well-being among adolescents on four consecutive time intervals of one year, where the age of the subjects at the first measurement is 12 years[1]. If the basis function coefficient of the first measurement occasion is constrained to zero, this formally means that the processes of assertiveness and well-being start at age 12. Another growth model for assertiveness and emotional well-being is the model in which both processes start at the birth of an individual. To incorporate this initial status into the growth model, the basis function coefficients for the growth rate factor have to be constrained to 12, 13, 14 and 15, successively. The measures of x, and y, now indicate the status of both processes twelve years after the origin of the process, if it may be assumed, at least, that the processes remain linear outside the domain of the time points measured. In other words, the true initial status of each process is shifted on the time axis. The origin of the process will be less, however, if there is no

[1] Mehta and West (2000) provide solutions when the age of the subjects is not equal at the first measurement occasion.

natural origin, and researchers have to define the origin of the process themselves, with the risk of misspecification.

Changing the Basis Function: the Lack of Invariance of the Growth Parameters to Transformations of the Time Scale

The effect of different time scales on the growth parameters is demonstrated by the transformations of the time scales t_{ix} and t_{iy}, defined by [$t_1=0$, $t_2=1$, $t_3=2$, $t_4=3$), into new time scales t_{ix}^* and t_{ix}^*, respectively by a linear function. Since we elaborate in this paper on the transformation of the time scale in one of the two growth processes, we will merely show the effect of the transformation in the y-side of the growth model. However, transformation of the time scale in the x-side of the model will have similar effects. The exposition is based on Garst (2000) and we will use the transformation function

$$t_{iy}^* = \alpha_y + \beta_y t_{iy} \tag{7}$$

In Function (7), α_y represents a shift on the time axis, and β_y is the scaling factor that represents a change in the time units, for example, from years to months. The transformation of t_{iy} induces to a transformation of the basis function of Λ_y into Λ_y^*:

$$\begin{bmatrix} 1 & t_{1y} \\ 1 & t_{2y} \\ 1 & t_{3y} \\ 1 & t_{4y} \end{bmatrix} \begin{bmatrix} 1 & \alpha_y \\ 0 & \beta_y \end{bmatrix} = \begin{bmatrix} 1 & \alpha_y + \beta_y t_{1y} \\ 1 & \alpha_y + \beta_y t_{2y} \\ 1 & \alpha_y + \beta_y t_{3y} \\ 1 & \alpha_y + \beta_y t_{4y} \end{bmatrix} = \Lambda_y^* \tag{8}$$

If $\quad \mathbf{P} = \begin{bmatrix} 1 & \alpha_y \\ 0 & \beta_y \end{bmatrix} \quad$ than

$$\mathbf{P}^{-1} = \begin{bmatrix} 1 & -\dfrac{\alpha_y}{\beta_y} \\ 0 & \dfrac{1}{\beta_y} \end{bmatrix} \quad \text{and} \quad \mathbf{\Lambda_y} = \mathbf{\Lambda_y^*}\ \mathbf{P^{-1}} \tag{9}$$

The implied covariance matrix $\mathbf{\Sigma}$ of Equation 3 than yields:

$$\Sigma = \begin{pmatrix} \Lambda^*_y P^{-1}(\Gamma\Phi\Gamma' + \Psi)P^{-1'}\Lambda^{*'}_y + \Theta_\varepsilon & \Lambda^*_y P^{-1}\Gamma\Phi\Lambda'_x \\ \Lambda_x \Phi\Gamma' P^{-1'}\Lambda^{*'}_y & \Lambda_x\Phi\Lambda'_x + \Theta_\delta \end{pmatrix} \qquad (10)$$

Because the transformation is only applied in the y-side of the model, the upper left quadrant of this matrix provides the interesting parameters. It shows how the transformation **P** of the basis function is absorbed by the transformation $\mathbf{P}^{-1}$ of the matrix $[\mathbf{\Gamma\Phi\Gamma'+\Psi}]$. We elaborate further on this matrix.

$\mathbf{P}^{-1}(\mathbf{\Gamma\Phi\Gamma'+\Psi})\,\mathbf{P}^{-1'}=$

$$= \begin{bmatrix} 1 & -\frac{\alpha_y}{\beta_y} \\ 0 & \frac{1}{\beta_y} \end{bmatrix} \begin{bmatrix} \gamma_{00} & \gamma_{01} \\ \gamma_{10} & \gamma_{11} \end{bmatrix} \begin{bmatrix} \varphi_{11} & \varphi_{21} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \begin{bmatrix} \gamma_{00} & \gamma_{10} \\ \gamma_{01} & \gamma_{11} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{\alpha_y}{\beta_y} & \frac{1}{\beta_y} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & -\frac{\alpha_y}{\beta_y} \\ 0 & \frac{1}{\beta_y} \end{bmatrix} \begin{bmatrix} \psi_{11} & \psi_{21} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{\alpha_y}{\beta_y} & \frac{1}{\beta_y} \end{bmatrix}$$

$$= \begin{bmatrix} \gamma_{00} - \frac{\alpha_y}{\beta_y}\gamma_{10} & \gamma_{01} - \frac{\alpha_y}{\beta_y}\gamma_{11} \\ \frac{1}{\beta_y}\gamma_{10} & \frac{1}{\beta_y}\gamma_{11} \end{bmatrix} \varphi_{11} \begin{bmatrix} \gamma_{00} - \frac{\alpha_y}{\beta_y}\gamma_{10} & \frac{1}{\beta_y}\gamma_{10} \\ \gamma_{01} - \frac{\alpha_y}{\beta_y}\gamma_{11} & \frac{1}{\beta_y}\gamma_{11} \end{bmatrix}$$

$$+ \begin{bmatrix} \psi_{11} - \frac{2\alpha_y}{\beta}\psi_{21} + \frac{\alpha_y^2}{\beta_y^2}\psi_{22} & \frac{1}{\beta_y}\psi_{21} - \frac{\alpha_y}{\beta_y^2}\psi_{22} \\ \frac{1}{\beta_y}\psi_{21} - \frac{\alpha_y}{\beta_y^2}\psi_{22} & \frac{\psi_{22}}{\beta_y^2} \end{bmatrix} \qquad (11)$$

Equation 11 makes clear that the parameters in the y-side of the growth model change as a function of a transformation of the metric of the time scale. We are now able to express the transformed parameters as a function of the original estimates:

$$\gamma_{00}^{*} = \gamma_{00} - \frac{\alpha_y}{\beta_y}\gamma_{10} \quad (12)$$

$$\gamma_{01}^{*} = \gamma_{01} - \frac{\alpha_y}{\beta_y}\gamma_{11} \quad (13)$$

$$\gamma_{10}^{*} = \frac{1}{\beta_y}\gamma_{10} \quad (14)$$

$$\gamma_{11}^{*} = \frac{1}{\beta_y}\gamma_{11} \quad (15)$$

$$\psi_{11}^{*} = \psi_{11} - \frac{2\alpha_y}{\beta_y}\psi_{21} + \frac{\alpha_y^{2}}{\beta_y^{2}}\psi_{22} \quad (16)$$

$$\psi_{21}^{*} = \frac{1}{\beta_y}\psi_{21} - \frac{\alpha_y}{\beta_y^{2}}\psi_{22} \quad (17)$$

$$\psi_{22}^{*} = \frac{\psi_{22}}{\beta_y^{2}} \quad (18)$$

of which Equation 16 to 18 also appear in Garst (2000), and Equation 12 to 15 in Stoel and Wittenboer (2000). From Equation 12 to 18 we may conclude for the y-side of the model that:

1) Multiplying the basis function of one growth process with a constant ß$_y$ leads to a change in all relevant parameters.
2) 2. If a constant α_y has been added to the basis function, changes will be found in the variance of the initial status and the covariance between initial status and growth rate of this specific growth process. Effects of the parameters of the other growth process on initial status are also subject to change. So, in addition to the changing relation between initial status and growth, the relationship between other growth processes and initial status is also affected by 'shifts' of the time scale.

3) The growth parameters are only dependent on the time scale, if the variance of the growth rate is nonzero between subjects (Garst, 2000, p. 64; Rovine & Molenaar, 1998). If there is a nonzero variance of the growth rate, than all growth parameters change proportionally, if the measurement unit ($ß_y$)of the time factor is subject to change. On the other hand, if the origin of the process (α_y) changes, only some of the growth parameters obtain other values. Since the change of α_y is potentially more dangerous in applied settings than the change of $ß_y$ because of its confusing nature, we will focus on this type of change and hold $ß_y$ equal to 1.
4) The correlation between initial status and growth rate will be equal to zero in a specific point in time t_0 (Rogosa & Willet, 1985). This point is defined by the expression $\alpha_y^0 = \psi_{21} / \psi_{22}$, where α_y^0 equals the number of time units between the origin of the process and the first measurement occasion (Stoel and Wittenboer, 2000).

Illustration at Empirical Data

The change in the growth parameters will be illustrated at data taken from the National Longitudinal Survey of Youth (NLSY) of Labor Market Experience in Youth. This study was initiated in 1979 by the U.S. Department of Labor to investigate the transition of young people into the Labor force and a detailed description of the data and data collecting procedures can be found in Baker, Keck, Mott and Quinlan (1993), and Curran (1997). The data used here are from a battery of assessments of Curran (1997).[2] In total 221 children had complete records on the four consecutive measures of 'antisocial behavior' and ‘reading recognition' (1986, 1988, 1990 & 1992). Antisocial behavior was measured by the mother's report on six items that assessed the child's antisocial behavior over the previous three month time period. The scale score for antisocial behavior could range in value from zero to 12. The reading recognition test measured word recognition and pronunciation ability; components considered to be essential to reading achievement. The scale score for reading recognition could range in value from zero to 8.4. The measurement unit was a two years period. The covariance matrix and the sample means of the four measurement occasions an antisocial behavior and reading recognition can be found in Table 1. The fact that List-wise deletion has been used has implications for the generalization of the results.

The models are tested to the covariance matrix and mean vector of Table 1 using M*plus* 1.04 (Muthén & Muthén, 1998). The model tested is similar to the model depicted in Figure 1. However, preliminary analyses lead to a rejection

2 The data can be found at http://www.unc.edu/-curran.

of the assumption of linear growth for the model concerning the outcome variable `reading recognition'. In the final multivariate model the Basis function coefficients for the third and fourth measures of reading recognition are therefore left free to estimate, and they represent the non-linear growth (Meredith & Tisak, 1990; Rovine & Molenaar, 1998).

Table 1: Sample means and covariance matrix of antisocial behavior (t_1- t_4) and reading recognition (t_1- t_4).

	Antisocial behavior				Reading reconition			
	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
Means Cov. matrix	1.493	1.837	1.878	2.068	2.516	4.041	5.021	5.803
t_1	2.369							
t_2	1.158	3.210						
t_3	1.224	1.630	3.244					
t_4	1.348	2.002	2.240	4.345				
t_1	-.122	-.041	-.064	-.205	.773			
t_2	-.228	-.170	-.213	-.253	.548	1.008		
t_3	-.341	-.215	-.279	-.232	.518	.894	1.217	
t_4	-.450	-.394	-.389	-.300	.492	.919	1.095	1.480

Note. N= 221; Source: Curran (1997)

The setup for the multivariate model is presented in Appendix A, and Table 2 Shows the different results when the values of the basis function for antisocial behavior are changed systematically. It contains the relevant parameter estimates of four fitted models with different specifications of the time scale in the basis function of antisocial behavior (λ_{iy}) Since the growth models yield the same expected covariance matrix (Stoel & Wittenboer, 2000), they give exactly the Same fit measures [χ^2 (20, N = 221) =17.17, p= .64; RMSEA= .00]. The difference between the models appears in the specification of the basis function for ‘antisocial behavior' (λ_{iy}).

Model 1 assumes that the growth process starts at the time of the first measurement occasion. The other models are obtained by adding (or subtracting) a constant to represent an origin after, or prior to the first measurement occasion (model 2, 3 and 4). As can be seen from Equations 12 to 28, the growth parameters must change if the basis functions are transformed to other values. Corresponding changes can be observed in the relevant growth parameters of antisocial behavior in Table 2: (1) the variance of the initial status (ψ_{11}) (2) the covariance between initial status and growth rate (ψ_{21}) (3) the effects of the growth parameters of reading recognition on the initial status and growth rate of antisocial behavior ($\gamma_{00,}$ γ_{10} and γ_{01}). Table 2 also contains the expectations of the initial status and growth rate of both processes (κ_0, κ_1, α_0 and α_1). As one can see, the mean initial status of antisocial behavior α_0 changes as well.

Table 2: Maximum Likelihood estimates of the parameters of the fitted growth models with different basis functions for antisocial behavior

Parameter	Modell [0, 1, 2, 3]	Model 2 [1, 2, 3, 4]	Model 3 [1.78, 2.78, 3.78, 4.78]	Model 4 [10, 11, 12, 13]
ϕ_{11}	.542 (6.79)	.542 (6.79)	.542 (6.79)	.542 (6.79)
ϕ_{22}	.160 (5.29)	.160 (5.29)	.160 (5.29)	.160 (5.29)
ϕ_{22}	.005 (.14)	.005 (.14)	.005 (.14)	.005 (.14)
ψ_{11}	.792 (3.51)	.547 (1.34)	.489 (.81)	6.980 (1.05)
ψ_{22}	.096 (1.82)	.096 (1.82)	.096 (1.82)	.096 (1.82)
ψ_{21}	.171 (2.00)	.075 (.58)	.000 (.00)	-.789 (-1.34)
γ_{00}	-.082 (-.53)	-.021 (-.11)	.027 (.12)	.531 (.687)
γ_{10}	-.061 (-.87)	-.061 (-.87)	-.061 (-.87)	-.061 (-.87)
γ_{01}	-1.071 (-3.37)	-1.287 (-3.27)	-1.456 (-3.07)	-3.233 (-2.08)
γ_{11}	.216 (1.54)	.216 (1.54)	.216 (1.54)	.216 (1.54)
κ_0	2.511 (41.37)	2.511 (41.37)	2.511 (41.37)	2.511 (41.37)
κ_1	1.537 (28.71)	1.537 (28.71)	1.537 (28.71)	1.537 (28.71)
α_0	3.397 (6.64)	3.397 (5.38)	3.396 (4.46)	3.394 (1.33)
α_1	.000 (.00)	.000 (.00)	.000 (.00)	.000 (.00)

Notes: Residuals variances are invariant across the 4 models [ε_1= 1.50 6.57]; ε_2=1.64 (9.37); ε_3= 1.36 (8.25) ; ε_4 = 1.43(5.71)]. The basis function for reading recognition was estimated at [0, 1, 1.63, 2.14]. The first element of the basis function equals α. Estimate/ standard error is given in the parentheses.

Parameter estimates for the base model with the origin of the process (the true initial status) defined at the first measurement occasion can be found under Model 1 with the specification [0, 1, 2, 3]. These parameters have the most straightforward interpretation, since the first measurement occasion corresponds with the true initial status and no assumption is needed for the growth curve outside the observed time interval. The specification of Model 1 leads statistically the least complicated model, and with respect to subject matter it has the simplest interpretation.

The specification in the basis function of Model 2 represents the usual setting in longitudinal multilevel analyses (Bryk & Raudenbush, 1987; Goldstein, 1995). Given the parameter estimates of Model 1 t_y^0 can be computed using $\alpha_y^0 = \psi_{21} / \psi_{21} = .171/.096 = 1.78$. This implicates that t_y^0 is located 1.78 time units prior to the first measurement occasion (Model 3). Model 3 on the contrary, has no 'natural' interpretation. It consists of a time point specification for which the variance of the initial status is minimal and the covariance between the initial status and growth rate of antisocial behavior is zero. Model 4 consists of an extreme, but realistic, time scale with the initial status defined at 10 time units prior to the first measurement occasion. This leads to settings of the basis function equal to [10, 11, 12, 14]. It becomes clear from Table 2 that the variance of the initial status ψ_{11} increases if the initial status is further and further apart from the minimum value at t_y^0 the covariance and expectation of the initial status change from positive to negative; γ_{00} becomes increasingly positive and γ_{01} becomes increasingly negative. As one can see in Model 3 (with the zero correlation between initial status and growth), the variance of initial status reaches a minimum value.

If one had interpreted the results based on Model 1, the following conclusions would have been drawn. The mean true growth curve for reading recognition is a non-linear curve with an initial status κ_0 = 2.511 and a growth rate κ_1 = 1.537. The partly estimated basis function [0, 1, 1.63, 2.14] reveals that the mean growth tempers as time continues. The variance of initial status (ϕ_{11} .542) and growth rate (ϕ_{22} .160) are both significant, but there is a nonsignificant covariance between initial status and growth rate ϕ_{21} .005). Eliminating the effects of the growth parameters of reading recognition on the growth process of antisocial behavior, a mean true growth curve remains for antisocial behavior with an initial status α_0 =3.497 and a growth rate α_1 = .000, with significant variation in initial status (ψ_{11} = .792), nonsignificant variation in growth rate (ψ_{22} = .096), and a significant positive covariance between initial status and growth rate (ψ_{21} .171). Thus, after controlling for effects of reading recognition, subjects having a higher initial status, also grow more in their antisocial behavior. From the reading recognition parameters, only the growth rate of reading recognition has a significant (negative) effect on the initial status of antisocial behavior (γ_{01} = -1.071). Thus, subjects growing faster on reading recognition would have a lower initial status of antisocial behavior.

Although these conclusions appear to be clear, they may change drastically if another specification of the time scale would have been used. Model 4, for instance, would have lead to a negative covariance between initial status and growth and a positive effect of the growth rate of reading recognition on the initial status of antisocial behavior ψ_{21}=-.789; γ_{01}= .531).

Discussion

The analysis in this paper Shows explicitly that the effects of growth parameters from one process to the growth parameters of another depend on the time scale being selected in the multivariate growth curve model. Although maximum likelihood estimation is scale-free (Long, 1984, p.58; Bollen, 1989, p.109) so that the expected covariance matrix will be invariant under linear transformations, the parameter estimates will not. Any other time scale, i.e. any linear transformation of a time scale, corresponds to parameter changes of the parameters involved and causes similar interpretation problems as in the area of multiple regression analysis (Aiken & West, 1991; Mehta & West, 2000).

All growth parameter results, being multivariate or not, depend on the time scale used. So, if the scale is arbitrary as in much of the social research, the results have an arbitrary interpretation as well; an interpretation that will be conditional on the selected time for the initial status. Confining us to shifts on the time scale (adding a constant), the change in covariance between the initial status and growth rate of antisocial behavior in Model 1 and Model 4 from .171 to -.789 is revealing. If no strong substantial argument prevails for the choice of the initial status, the parameter estimates just reflect arbitrarily selected levels of the process, and it will be much better, then, to use the term 'level' instead of `initial status' (McArlde & Hamagani, 1991; Rovine & Molenaar, 1998). Note, however, that the growth rate is invariant across models.

Problems get worse, if the time unit changes as well. For instance, if we measure in years instead of decades and we take the first measurement occasion as initial status. In that case, both growth rate and initial status are subject to change and the interpretation becomes more complicated. Being merely interested in pure change, on the other hand, we simply need confine ourselves to the same unit of time and we need not bother about time scale shifts by adding constants. Growth rates of processes may be compared in that case, for example between groups or between different processes in the same multivariate model, because growth rates are insensitive to adding a constant to the time scale. If the time unit does not change, the growth rate does not change either. In other words, growth rates are invariant under shifts on the time scale.

Apparently, problems arise if researchers overreach their goals in interpreting the initial status, or more specifically de correlation between initial status and growth rate. As is shown in Stoel & Van den Wittenboer (2000) already, this is meaningful only in those cases in which the processes have a natural origin; if time is measured on a ratio scale, so to speak. Although specific situations exist in which the origin of the process is known, and the correlation between initial status and growth rate can be substantively interpreted, most social science research is limited to growth per se.

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Appendix A

In this appendix we present the M*plus* 1.04 programs that were used to fit Models 1 through 4. Since all have a similar structure, we present the program used to fit Model 1 in its entirety. The remaining models can be fitted by changing the restrictions imposed on the slope factor of antisocial behavior (SlopeA) according to Table 2. For further information on the program specification we refer to the M*plus* User's Guide (Mutken & Mutken, 1998).

```
TITLE: multivariate growth model on Curran data
DATA: file is currandata.dat;
VARIABLE:
Names are anti 1 anti2 anti3 anti4 read1 read2 read3 read4;
usevariables are anti1 anti2 anti3 anti4 read1 read2 read3 read4;
ANALYSIS:
type = meanstructure,
MODEL:
InterA by anti1-anti4@1;
SlopeA by anti 1 @0 anti2@1 1 anti3@2 anti4@3;
InterR by read1-read4@1;
SlopeR by read1@0 read2@1 read3 read4;
InterA on InterR SlopeR;
SlopeA on InterR SlopeR;
InterA with SlopeA;
InterR with SlopeR;
[anti1-read4@0 InterR SlopeR InterA SlopeA];
```

Below we also present the program that can be used to fit the models using the LISREL 8.30 program (Jöreskog & Sörbom, 2000).

```
Multivariate growth model on Curran data
DA ni=8 no=221 ma=cm
RA fi=currandata.dat
LA
Anti1 anti2 anti3 anti4 read1 read2 read3 read4
SE
Read1 read2 read3 read4 anti1 anti2 anti3 anti4
MO ny=8 ne=4 ly=fu,fi te=di,fr ps=sy,fi be=fu,fi ty=ze al=fr
LE
InterR SlopeR InterA SlopeA
MA ly
```

1000
1100
1000
1000
0010
0011
0012
0013
Fr ly 3 2 ly 4 2
Fr ps 1 1 ps 2 2 ps 3 3 ps 4 4 ps 2 1 ps 4 3
Fr be 3 1 be 4 1 be 3 2 be 4 2
OU se rs ad=off sc ss